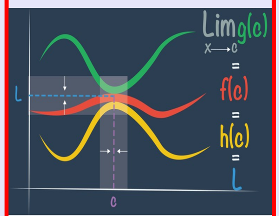
Calculus I

Lecture 50



Feb 19-8:47 AM

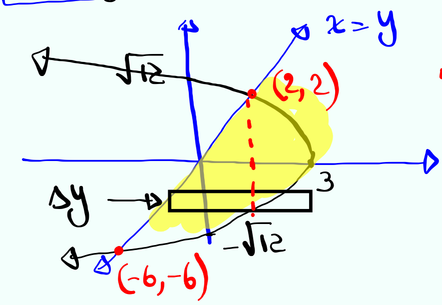
Draw the region bounded by $\boxed{x=y}$ and $\boxed{4x+y^2=12}$, then find its area. $\to 4x=-y^2+12$

$x=-\frac{1}{4}y^2+3$

Parabola opens left

x-Int $(3,0)$

Y-Int $(0,\pm\sqrt{12})$



$4x+y^2=12$

$4y+y^2=12$

$y^2+4y-12=0$

$(y-2)(y+6)=0$

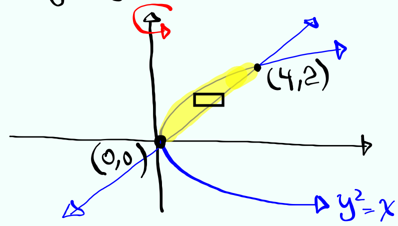
$y=2 \quad y=-6$

$$A=\int_{-6}^{2}(R-L)\,dy=\int_{-6}^{2}\left(-\frac{1}{4}y^2+3-y\right)dy$$

$$=\left(-\frac{1}{4}\cdot\frac{y^3}{3}+3y-\frac{y^2}{2}\right)\Bigg|_{-6}^{2}=\square$$

Dec 3-7:27 AM

Rotate the region bounded by $y^2 = x$ and $x = 2y$ by the Y-axis, then find its volume.



$$\begin{cases} y^2 = x \\ x = 2y \end{cases}$$

$$y^2 = 2y$$
$$y^2 - 2y = 0$$
$$y(y-2) = 0$$
$$y = 0 \;,\; y = 2$$
$$x = 0 \;,\; x = 4$$

Cannot use disk Method since ▯ is not ⊥ axis of Rev.

with the new Ref. Rect ⊥ axis of revolution however region is not completely attach to A.O.R.

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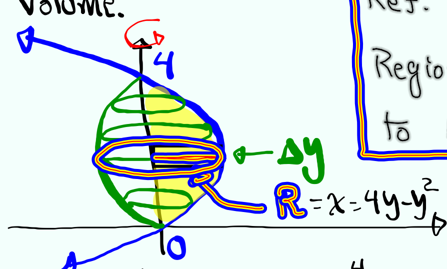
we can do this by washer Method or Shell Method.

Dec 3-7:37 AM

Rotate the region bounded by y-axis and $x = 4y - y^2$ about the Y-axis, then find the Volume.



Ref. Rec. ⊥ A.O.R.
Region totally attached to A.O.R.

Disk Method

$$V = \int_0^4 \pi (R(y))^2 dy = \int_0^4 \pi (4y - y^2)^2 dy$$

$$= \pi \int_0^4 \left[16y^2 - 8y^3 + y^4\right] dy$$

$$= \pi \left[\frac{16y^3}{3} - \frac{8y^4}{4} + \frac{y^5}{5}\right]\Bigg|_0^4 = \boxed{}$$

Dec 3-7:46 AM

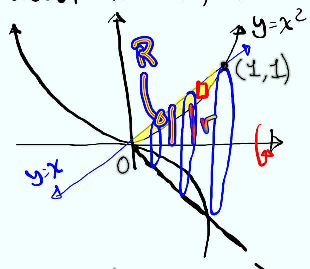
When to use washer Method

1) Ref. Rect. $\perp$ A.O.R.

2) Region is not totally attached to A.O.R

$$V = \int_a^b \pi\left[R^2 - r^2\right] d?$$

Rotate the region bounded by $y=x^2$ & $y=x$ about x-axis, then find the volume.



1) Ref. Rect. $\perp$ A.O.R.

2) Region not totally attached to A.O.R.

washer Method

$R = x$ $\quad$ $r = x^2$

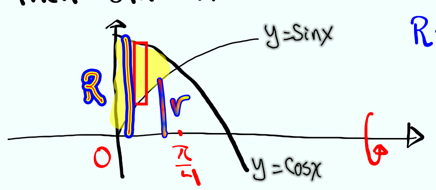
$$V = \int_0^1 \pi\left[x^2 - (x^2)^2\right]dx = \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]\Big|_0^1 = \boxed{\frac{2\pi}{15}}$$

Dec 3-7:54 AM

Rotate the region bounded by $y=\sin x$, $y=\cos x$, $x=0$, and $x=\frac{\pi}{4}$ about x-axis, then find its Volume.

washer

$R = \cos x$ $\quad$ $r = \sin x$



$$V = \int_0^{\pi/4} \pi\left[\underbrace{\cos^2 x - \sin^2 x}\right]dx = \pi\int_0^{\pi/4} \cos 2x \, dx$$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$$= \pi\int_0^{\pi/2} \cos u \, \frac{du}{2}$$

$$= \frac{\pi}{2} \sin u \Big|_0^{\pi/2} = \frac{\pi}{2}\left[\sin\frac{\pi}{2} - \sin 0\right] = \boxed{\frac{\pi}{2}}$$

$x=0 \quad u=0$

$x=\pi/4 \quad u=\pi/2$

Dec 3-8:04 AM

$$f(x) = \int_{\sqrt{x}}^{x^3} \cos t^2 \, dt$$

$$f(x) = \int_{u(x)}^{v(x)} g(t) \, dt$$

find $f'(x)$

$$f'(x) = g(v(x)) \cdot v'(x) - g(u(x)) \cdot u'(x)$$

$$f'(x) = \cos(x^3)^2 \cdot \frac{d}{dx}[x^3] - \cos(\sqrt{x})^2 \cdot \frac{d}{dx}[\sqrt{x}]$$

$$= \cos x^6 \cdot 3x^2 - \cos x \cdot \frac{1}{2\sqrt{x}}$$

$$= 3x^2 \cos x^6 - \frac{1}{2\sqrt{x}} \cos x$$

Dec 3-8:15 AM

$$f(x) = \int_0^x \underbrace{\frac{t^2}{t^2+t+2}}_{g(t)} dt$$

$v(x)$ $u(x)$

Discuss increasing, decreasing, concave up, and concave down.

$$f'(x) = \frac{x^2}{x^2+x+2} \cdot \cancel{\frac{d}{dx}[x]}^{1} - \frac{0^2}{0^2+0+2} \cdot \cancel{\frac{d}{dx}[0]}^{0}$$

$$f'(x) = \frac{x^2}{x^2+x+2} = \frac{x^2}{x^2+x+\frac{1}{4} - \frac{1}{4}+2} = \frac{x^2}{(x+\frac{1}{2})^2 + \frac{7}{4}}$$

$f'(x) > 0 \rightarrow f(x)$ increasing

$$f''(x) = \frac{2x(x^2+x+2) - x^2(2x+1)}{(x^2+x+2)^2} = \frac{\cancel{2x^3}+2x^2+4x-\cancel{2x^3}-x^2}{(x^2+x+2)^2}$$

$$f''(x) = \frac{\overset{x(x+4)}{x^2+4x}}{(x^2+x+2)^2}$$

$f''(x) = 0$

$x^2+4x = 0$

$x(x+4) = 0$

$x = 0 \quad x = -4$

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C.U.	−4	C.D.	0	C.U.

Dec 3-8:21 AM